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THE INVERSE DIFFRACTION PROBLEM: ANALYSIS OF SPECULARS

JUNE 1967

R. M. Lewis (Consultant)

Prepared for  
SPACE DEFENSE SYSTEMS PROGRAM OFFICE  
DEPUTY FOR SURVEILLANCE AND CONTROL SYSTEMS  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
L. G. Hanscom Field, Bedford, Massachusetts



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Prepared by

THE MITRE CORPORATION  
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## ABSTRACT

Specular returns from radar targets contain large power which makes a technique based on the analysis of speculars attractive. Herein, the specular returns from a flat region and from a smooth surface tangent to a plane along a curve, which are the largest returns for reasonably sized bodies, are used to obtain information about the geometry of the target.

## REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

A handwritten signature in cursive script that reads "Thomas O. Wear". The signature is written in dark ink and is positioned above the printed name and title.

THOMAS O. WEAR, Colonel, USAF  
Director, Space Defense Systems Program Office  
Deputy for Surveillance and Control Systems



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## INTRODUCTION

Specular returns from scattering targets may be classified according to the order of magnitude of the return. It can be shown that some typical returns are (after suitable normalization) of the following order in  $ka$ , where  $k$  is the wave-number and  $a$  is a typical target dimension:

<u>Order</u>	<u>Surface Geometry</u>
$O \left[ (ka)^2 \right]$	flat region
$O \left[ (ka)^{3/2} \right]$	smooth surface tangent to a plane along a curve
$O \left[ (ka) \right]$	Convex surface with non-zero Gaussian curvature
$O \left[ (ka)^{1/2} \right]$	edge-diffraction
$O \left[ 1 \right]$	vertex-diffraction

Because of the large power of a specular return it would be convenient to use it to obtain information about the geometry of the corresponding specular region. In this paper we develop a method for doing this for the first two types of surface regions in the above table.

## QUASI-MONOSTATIC SCATTERING

Let  $\underline{I}$  be a unit vector in the direction of propagation of the plane wave

$$u_i = e^{ik\underline{I} \cdot \underline{X}}, \quad (1)$$

which is incident on a target. If the total (scalar) field satisfies the boundary condition  $u = 0$  on the surface of the target, then according to the Kirchhoff (physical optics) approximation, the scattered far-field is given by (see Equation (2.25) of [1])

$$\mu_s = \frac{i e^{ikr}}{2 \pi k r} g, \quad (2)$$

where

$$g = -k^2 \iint_{\ell} \underline{I} \cdot \underline{N} e^{ik(\underline{I} - \underline{J}) \cdot \underline{X}} ds(\underline{X}). \quad (3)$$

Here  $k$  is the wave-number,  $r$  is the range.  $\underline{N}$  is the outward unit normal vector on the surface of the target,  $\underline{J}$  is a unit vector pointing from the target to the point of observation, and the surface integral of Equation (3) is taken over the illuminated portion  $\ell$  of the target. A similar formula holds for the boundary condition  $\partial u / \partial n = 0$ , and for scattering of an electromagnetic wave by a perfectly conducting target (see [1]).

A rectangular coordinate system such that  $\underline{I} = (0, 0, -1)$ ,  $\underline{J} = (\sin \theta, 0, \cos \theta)$ , and  $\underline{X} = (x, y, z)$  is introduced. In the quasi-monostatic case  $\theta \ll 1$  and  $\underline{J} \sim (\theta, 0, 1)$ . Then since  $-\underline{I} \cdot \underline{N} ds = dxdy$ , Equation (3) becomes

$$g(\theta) = k^2 \iint_P e^{-ik[\theta x + 2z(x, y)]} dxdy, \quad (4)$$

where  $P$  is the projection of  $\ell$  onto the  $xy$ -plane.

---

[1] R. M. Lewis, Notes on the Kirchhoff Method in Scattering Problems.

For simplicity it is now assumed that the region  $P$  is intersected, at most, twice by each line parallel to the  $y$ -axis. Then its boundary is given by the two functions  $y_1(x)$  and  $y_2(x)$  as illustrated in Figure 1. These functions are defined so that they are continuous and have constant values outside of  $P$ . (Then  $y_1 \equiv y_2$  outside of  $P$ ). We take  $2L$  greater than the largest diameter of  $P$  parallel to the  $x$ -axis and choose the origin so that the points  $\pm L$  lie outside of  $P$ .

From Equation (4) we now see that

$$g(\theta) = k \int_{-L}^L \gamma(x) e^{-1k\theta x} dx, \quad (5)$$

where

$$\gamma(x) = k \int_{y_1}^{y_2(x)} e^{-2ikz(x,y)} dy. \quad (6)$$

From the theory of Fourier series it is known that if  $f(x)$  is piecewise smooth and if the Fourier coefficients

$$a_n = \frac{1}{2L} \int_{-L}^L f(x) e^{-\frac{in\pi x}{L}} dx, \quad (7)$$

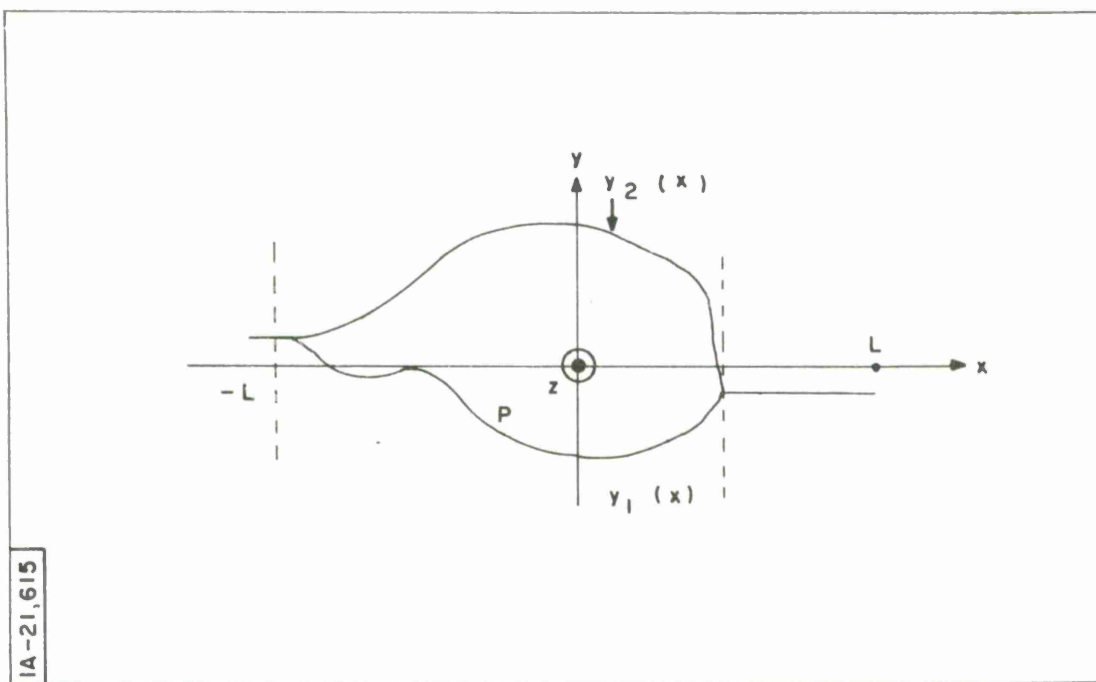


Figure 1. Boundary of P Given by Two Functions

are introduced, then

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{\frac{in\pi x}{L}} \quad (8)$$

Therefore, if

$$\theta = \frac{n\pi}{kL}, \quad n = 0, \pm 1, \pm 2, \dots \quad (9)$$

is set, then

$$\gamma(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{L}} \quad (10)$$

where

$$C_n = \frac{1}{2kL} g\left(\frac{n\pi}{kL}\right) \quad (11)$$

Thus if  $g$  can be measured for those values of  $\theta$  given by Equation (9)  $\gamma(x)$  can be determined. The latter function is closely related to the geometry of the target as illustrated in the next section. A multi-static system for making the required measurements instantaneously is illustrated in Figure 2 (other measuring systems can obviously be used).

As illustrated in Figure 2, receivers are placed on a straight line at distances from the transmitter that are an integral multiple of  $\tau$ . The system is characterized by four length parameters:

- $r$  range
- $\tau$  receiver spacing
- $2L$  body dimension (upper bound)
- $\lambda = 2\pi/k$  wave-length

Since  $r \sim n \tau$ , we see from Equation (9) that the four parameters must satisfy the condition

$$\frac{\tau}{r} = \frac{\lambda}{2L} . \quad (12)$$

In practice one would first choose  $2L$  larger than the maximum expected dimension of the target and then set  $\lambda = 2L\tau/r$ .

Of course it is impractical to measure  $C_n$  except for small  $n$ , say  $n = 0, \pm 1, \pm 2$ . Therefore, it is envisioned that measurements will be made only of specular returns. For such returns the larger Fourier coefficients will be negligible and a small number of terms in Equation (10) should give a very good approximation to  $\gamma(x)$ . This is certainly true if  $\gamma(x)$  is smooth. If not, then a few terms will yield a smoothed approximation to  $\gamma(x)$ .

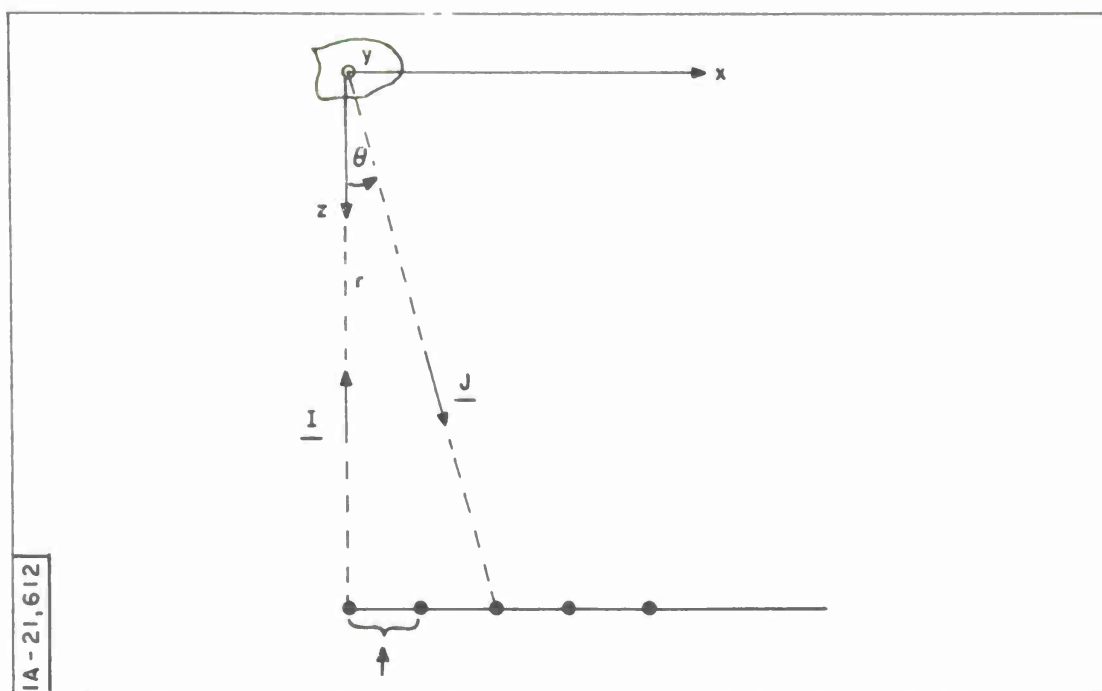


Figure 2. Illustration of a Multistatic System for Making Required Measurements Instantaneously

#### SPECULAR ANALYSIS OF A FLAT REGION

The largest specular returns are produced by flat portions of a target. Assume that the region  $P$  in Figure 1 is a flat surface region normal to the direction of incidence  $\underline{I}$ . Then one may take  $z(x,y) = 0$  in Equation (6). Now

$$\gamma(x) = kw(x) \quad (13)$$

where  $w$  is the width function

$$w(x) = y_2(x) - y_1(x). \quad (14)$$

Furthermore

$$g(0) = k \int_{-L}^L \gamma(x) dx = k^2 A, \quad (15)$$

where  $A$  is the area of  $P$ . Thus the monostatic specular return of a flat

region is of order  $(ka)^2$  as noted in the introduction. Furthermore, since  $\gamma(x)$  is real, one sees from Equation (5) that  $g^*(\theta) = g(-\theta)$ . Hence  $C_{-n} = C_n^*$ . Thus, for a flat plate, the number of receiver sites can be reduced by a factor of two.

The function  $w(x)$  does not uniquely determine the shape of the region  $P$ , i. e., the functions  $y_1(x)$  and  $y_2(x)$ , but we will see in the next section that if a second measurement is made corresponding to a rotation in the  $xy$ -plane, the shape of the region  $P$  can be found.

#### SOLUTION OF THE WIDTH PROBLEM

Let  $P$  be a region in the  $xy$ -plane whose boundary is intersected, at most, twice by each line parallel to the  $y$ -axis, as illustrated in Figure 1. We introduce a rotation

$$\xi = x \cos \alpha + y \sin \alpha, \eta = -x \sin \alpha + y \cos \alpha. \quad (16)$$

Then

$$y = \xi \sin \alpha + \eta \cos \alpha, x = \xi \cos \alpha - \eta \sin \alpha. \quad (17)$$

The rotated axes are illustrated in Figure 3.

Let  $x_1(\xi, \alpha)$  and  $x_2(\xi, \alpha)$  be the  $x$ -coordinates of the two boundary points on the dashed line  $\xi = \text{constant}$  parallel to the  $\eta$ -axis in Figure 3. From Equation (16) we see that the functions  $x_j(\xi, \alpha)$  are defined implicitly by

$$\xi = x_j \cos \alpha + y_j(x_j) \sin \alpha, j = 1, 2. \quad (18)$$

Let  $w(\xi, \alpha)$  be the length of the dashed line joining the two boundary points, i. e., the width of the region  $P$  in the direction of the  $\eta$ -axis. Then from Equation (16)

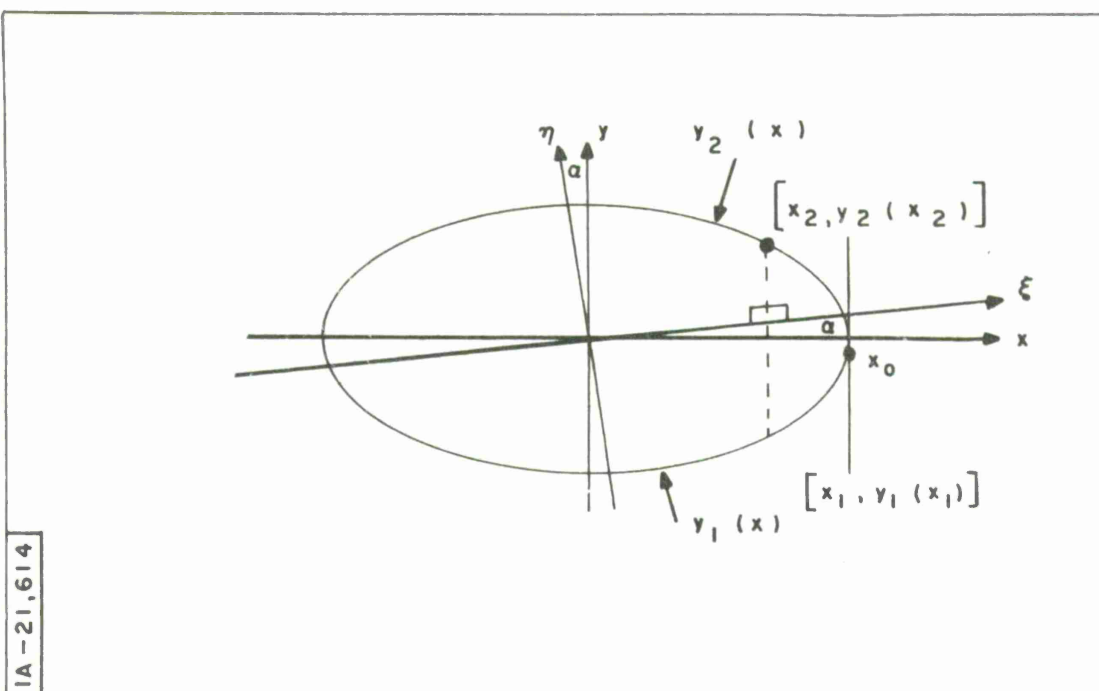


Figure 3. Rotated Axes

$$\begin{aligned}
 w(\xi, \alpha) &= \eta_2 - \eta_1 = [-x_2 \sin \alpha + y_2(x_2) \cos \alpha] - [-x_1 \sin \alpha + y_1(x_1) \cos \alpha] \\
 &= (x_1 - x_2) \sin \alpha + [y_2(x_2) - y_1(x_1)] \cos \alpha.
 \end{aligned} \tag{19}$$

The following width problem is considered: Given  $w(\xi, \alpha)$  for  $\alpha = 0$  and a second value of  $\alpha$  to determine the functions of  $y_1(x)$  and  $y_2(x)$ .

Actually the second value of  $\alpha$  very near  $\alpha = 0$  will be taken. More precisely, the function  $\omega_\alpha(\xi, 0)$  will be used.

At  $\alpha = 0$ , we see from Equation (18) that  $x_j = \xi$ , and by differentiating Equation (18) with respect to  $\alpha$  we obtain

$$\frac{\partial x_j}{\partial \alpha} = y_j(\xi). \tag{20}$$



It follows from Equation (19) that at  $\alpha = 0$

$$w(x, 0) = y_2(x) - y_1(x) \quad (21)$$

and from Equations (4) and (5) one finds that

$$w_{\alpha}(\xi, 0) = y_2'(\xi)[-y_2(\xi)] - y_1'(\xi)[-y_1(\xi)] = -\frac{1}{2} \frac{d}{d\xi} [y_2^2 - y_1^2] \quad (22)$$

Since (see Figure 3)

$$y_2(x_0) = y_1(x_0) \quad (23)$$

Equation (7) may be integrated to obtain

$$(y_2 + y_1)(y_2 - y_1) = y_2^2(x) - y_1^2(x) = q(x) \quad (24)$$

where

$$q(x) = -2 \int_{x_0}^x w_{\alpha}(\xi, 0) d\xi \quad (25)$$

From Equations (21) and (24) one sees that

$$y_2(x) + y_1(x) = \frac{q(x)}{w(x, 0)} \quad (26)$$

and Equations (21) and (26) are easily solved to yield

$$y_{1/2}(x) = \frac{1}{2} \left[ \frac{q(x)}{w(x, 0)} \mp w(x, 0) \right] \quad (27)$$

Thus Equations (25) and (27) provide a solution to the width problem.

# SPECULAR ANALYSIS OF A SMOOTH SURFACE WHICH IS TANGENT TO A PLANE ALONG A CURVE

As a second example a regular surface which is tangent to the plane  $z = 0$  along a curve  $y = y_0(x)$  is considered.\* Important examples are cylinders and cones; in these cases the curve is a straight line. The integral Equation (6) is evaluated by the method of stationary phase. A stationary point occurs at  $y = y_0(x)$ . If the phase function  $\phi(y) = -2z(x, y)$  is introduced, then at the stationary point  $\phi = 0$ ,  $\phi' = 0$ , and  $\phi'' = 2z_{yy}(x, y_0) > 0$ .

Hence

$$\gamma(x) \sim (\pi k)^{1/2} e^{i\pi/4} [-z_{yy}(x, y_0)]^{-1/2}. \quad (28)$$

But  $-z_{yy}$  is the curvature of the normal section of the surface in the direction of the  $y$ -axis. If we introduce the principal curvatures  $K_1 = 0$  and

$K_2 = K = 1/\rho$ , then it can be shown that  $-z_{yy} = K \cos^2 \beta$  where

$\tan \beta = y'_0(x)$  (see Figure 4).

It follows that

$$\gamma(x) \sim e^{i\pi/4} \sqrt{\pi k \rho [x, y_0(x)] [1 + (y'_0)^2]}. \quad (29)$$

Since the element of arc-length is  $ds = \sqrt{1 + (y'_0)^2} dx$  it follows that

$$g(0) = k \int_{-L}^L \gamma(x) dx \sim e^{i\pi/4} k \sqrt{\pi} \int \sqrt{k\rho} ds. \quad (30)$$

---

\* It can be shown (by using the formula of Rodrigues) that the Gaussian curvature vanishes at each point of the curve and that the curve is a principal curve corresponding to the principal curvature  $K = 0$ . Conversely if  $K \equiv 0$  along a principal curve, then that curve is a plane curve.

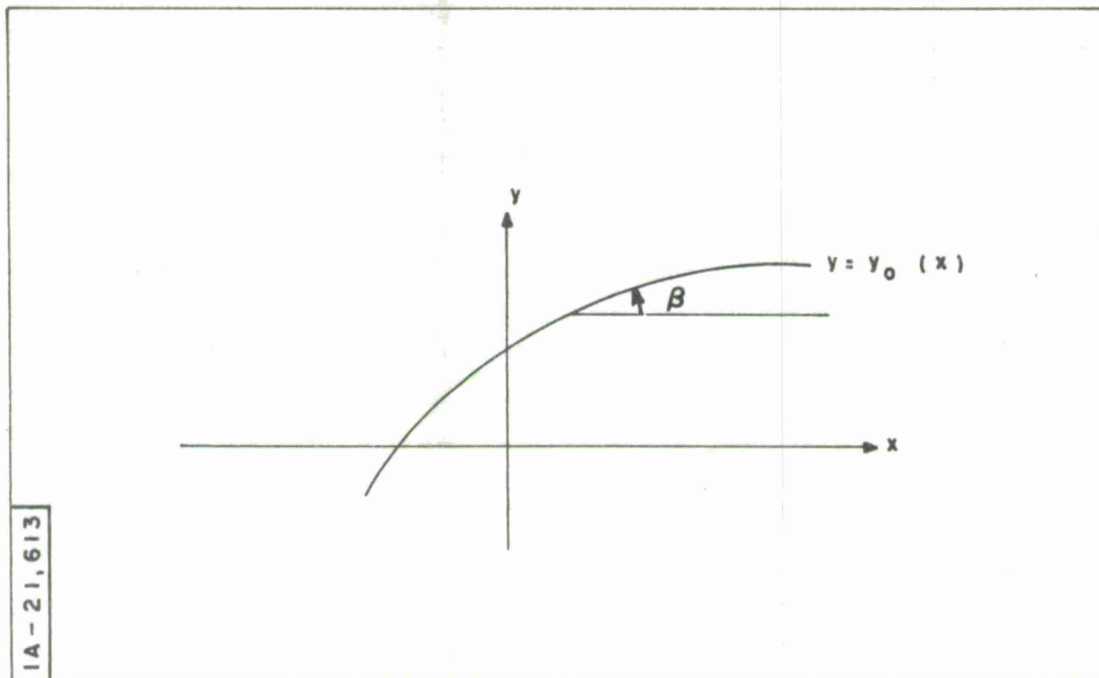
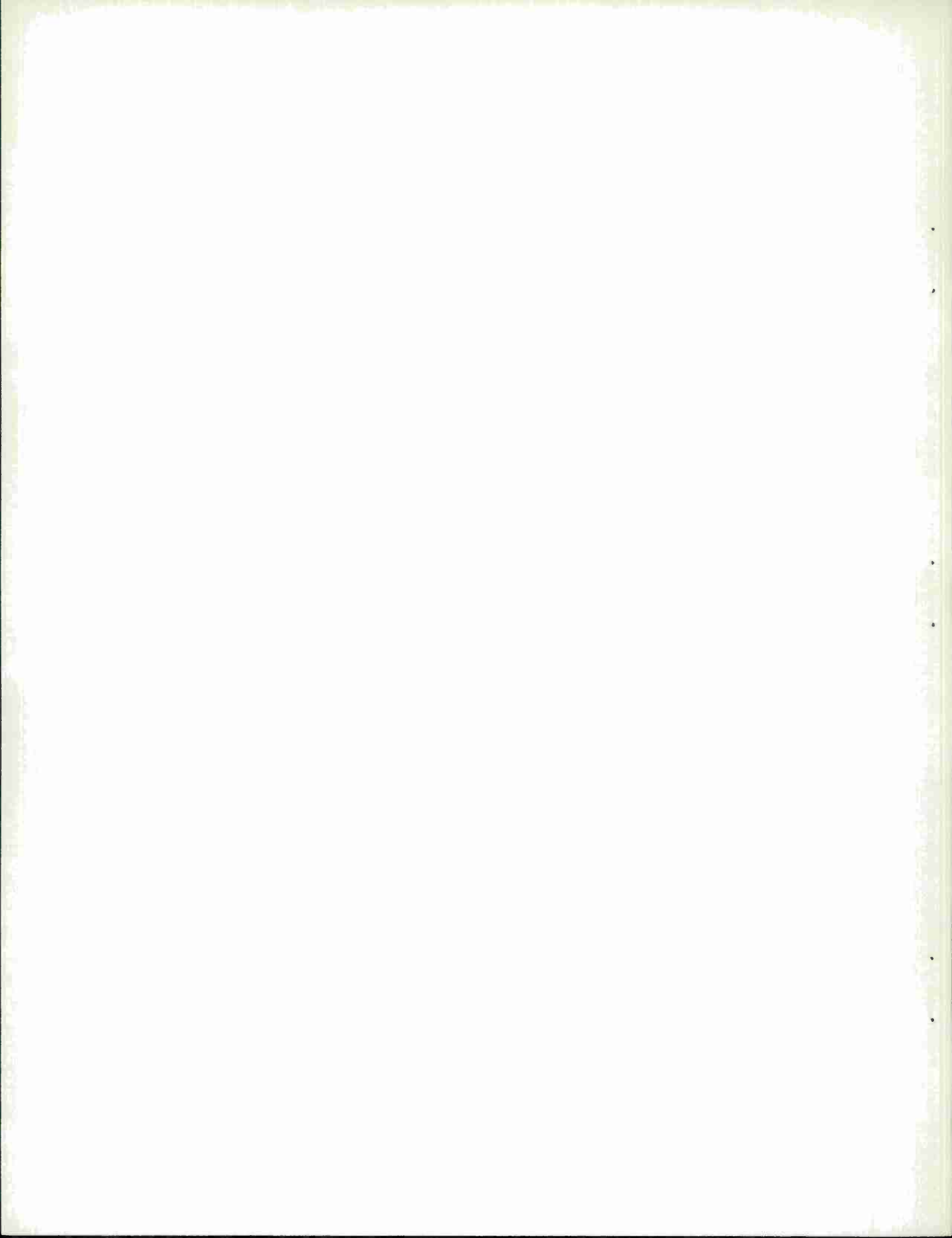


Figure 4. Smooth Surface Tangent to a Plane Along a Curve

Thus the monostatic specular return is of order  $(ka)^{3/2}$  as noted in the introduction.

We see from Equation (29) that  $\gamma(x)$  depends on the curve of tangency  $y = y_0(x)$  and the non-zero principal radius of curvature  $\rho$  along this curve. These two functions characterize the geometry of the specular region in this case just as the two functions  $y_1(x)$  and  $y_2(x)$  determine the shape of the flat region on pages 6 & 7. On pages 7-9 it was proved how to determine  $y_1$  and  $y_2$ , given  $\gamma$ . The analogous problem for the determination of  $y_0$  and  $\rho$ , given  $\gamma$ , remains to be considered.



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